A multi-criteria decision making approach for food engineering

Abakarov, A.[a]

[a]*Universidad Politécnica de Madrid, Madrid, Spain (alik.abakarov@upm.es)*

ABSTRACT

The objective of this study was to propose a decision making approach and tools (software packages) to solve the multi-criteria decision making problems arising in the food engineering. The proposed decision making approach is based on a simultaneous utilization for a given set of Pareto-optimal solutions the two following decision making methods: 1) well-known Analytic Hierarchy Process method and 2) Tabular Method. The using of Tabular Method allows utilizing the AHP method in a straightforward manner, which avoids the information overload and makes the decision making process easier. The aggregating functions approach, adaptive random search algorithm coupled with penalty functions approach, and the finite difference method with cubic spline approximation were utilized in this study to compute the initial set of the Pareto-optimal solutions. The decision making software "MPRIORITY" and "T-CHOICE" based on the Analytic Hierarchy Process and Tabular Method methods, respectively, were utilized for choosing the best alternative among the obtained set of Pareto-optimal solutions. The proposed in this study approach and tools was successfully tested on the multi-objective optimization problem of the thermal processing of packaged food. The proposed decision making approach and tools are useful for food scientists (research and education) and engineers (real thermal food process evaluation and optimization).

Keywords: multi-criteria optimization, Pareto-optimal solution, decision-making approach, food engineering needs, sophisticated software.

INTRODUCTION

It is well known that the majority of real-life optimization problems, including the problems arising in the food engineering, are of a multi-objective nature with conflicting objectives, where it is necessary to compute more than one non-dominated or Pareto-optimal solutions [1, 2]. Construction of the set of Pareto-optimal solutions is of primary importance in the above problems. Various multi-objective optimization approaches to construct the set of Pareto-optimal solutions have been proposed over the last few decades [1, 2, 3]. Several of these approaches already were successfully applied to the food engineering problems [1, 2, 3]. Each of the Pareto-optimal solutions can be considered as a final "compromise" solution of a multi-objective optimization (MOO) problem, .i.e. Pareto optimal solutions are regarded as equally desirable in the mathematical sense. Hence, it is necessary to identify the most preferred one among the Pareto optimal solutions. In order to do this various multi-criteria decision making approaches been proposed over the last few decades [4]. These approaches refer to the solving of decision problems involving multiple and conflicting goals, coming up with a final solution that represents a required compromise. In the field of food engineering, multi-criteria decision making approaches have received relatively little attention. Therefore, the main objective of this study was to propose an approach and decision support tools for solving the multi-criteria decision making problems arising in the food engineering. The multi-objective optimization problem of the thermal processing of packaged food was chosen to illustrate the applicability of the proposed approach.

MATERIALS & METHODS

Pareto-optimal solutions

A general multi-objective optimization (MOO) problem can be formulated as follows:

$$\Phi(x) = \langle f_1(x), f_2(x), \ldots, f_l(x) \rangle \to \min_{x \in X}, \tag{1}$$

where: $X \subset R^n$ is a non-empty set of feasible decisions (a proper subset of R^n), $x = \langle x_1, x_2, \ldots, x_n \rangle \in X$ is a real $n-$vector decision variable, and $f_i : R^n \to R$ are particular multi-objective functions. We assume that all of the constraints are included in the particular objective functions (1) by utilizing the penalty functions. If no vector $x^* = \langle x_1^*, x_2^*, \ldots, x_n^* \rangle \in X$ exists such that $x^* = \arg\min_{x \in X} f_i(x)$, $\forall i \in 1:l$, that is if no vector exists that is optimal for all objectives concurrently, then there is no unique optimal solution — if it exists we call such a solution a utopian solution — and a concept of acceptable solutions is needed. The subset $WP(X) = \{x^p \in X : \textit{such that there does not exist an } x \in X \textit{ with } f_i(x) \le f_i(x^p), \forall i \in 1:l\}$ is called the set of Pareto-optimal solutions of the problem (1). Pareto-optimal solutions are the only acceptable solutions of a multi-objective optimization problem, since any other solution can be improved. Pareto-optimal solutions are also known as non-dominated or efficient solutions. The space in E^l formed by the points of the set $P(X) = \{x \mid x \in WP(X)\}$ is called a Pareto optimal frontier or front. Figure 1 provides a visualization of the definitions made for the two-dimensional MOO problem (1) and two particular objectives.

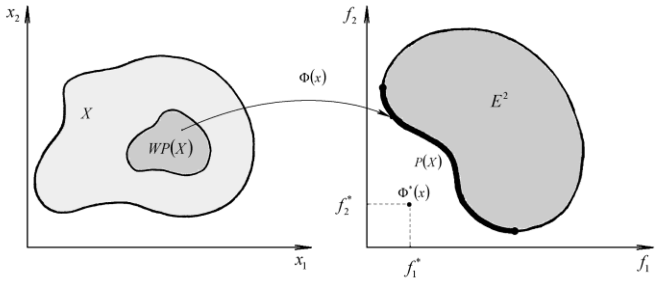

Figure 1. Visualization of 2-dimensional MOO problem and 2 particular objectives

Multi-objective optimization approach

The multi-objective optimization approach used in this study is based on optimizing the following aggregating functions by using the adaptive random search algorithm [1].

<u>Function 1.</u> Linear weighted sum aggregating function

$$\Phi(x) = \sum_{i=1}^{l} \lambda_i f_i(x) \to \min_{x \in X}, \ \sum_{i=1}^{l} \lambda_i = 1, \ \lambda_i \ge 0, \tag{2}$$

where λ_i is the weight used for the i-th particular objective function $f_i(x)$.

<u>Function 2.</u> Weighted min-max aggregating function

$$\Phi(x) = \min_{x \in X} \max_{i \in 1:l} \lambda_i f_i(x), \ \sum_{i=1}^{l} \lambda_i = 1, \ \lambda_i \ge 0. \tag{3}$$

<u>Function 3.</u> The penalty aggregating function,

$$\Phi(x) = f_k(x^s) + \sum_{j=1}^{l} P_j(x^s) \to \min_{x \in X}, \tag{4}$$

where $k, k \in 1:l$ is a randomly chosen number at the first step of an adaptive random search of a particular objective function, $f_k(x^s)$ is the value of the k-th particular objective function at step s of the adaptive random search algorithm, and $P_j(x^s)$ is the penalty function of the j-th particular objective function computed at step s of the adaptive random search algorithm. The following formula is used to compute the penalties $P_j(x^s), \ j \in 1:l$:

$$P_j(x^s) = A\left(\left|f_j(x^s) - f_j(x^{s-1})\right| + f_j(x^s) - f_j(x^{s-1})\right), \tag{5}$$

where A is a sufficiently large number.

Adaptive random search algorithm

The adaptive random search algorithm belongs to a specific class of global stochastic optimization algorithms [6]. This class of algorithms is based on generating the decision variables from a given probability distribution, and the term ‘‘adaptive” consists of modifications to the probability distribution utilized in the searching process, which, throughout the whole search process, act as minimum computations of the objective function, locating global solutions. The pedestal probability distribution is utilized in the adaptive random search. After every calculation of objective function, the pedestal distribution of decision variables is modified so that the probability of finding the optimal value of the objective function is increased. For example, Fig. 2 shows a pedestal frequency distribution for the two-dimensional case of an optimization problem can be obtained in the middle of the search process.

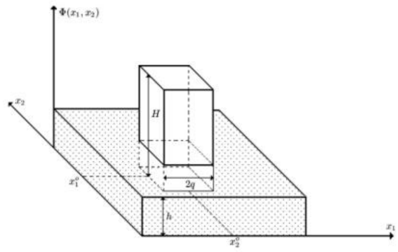

Figure 2. Pedestal frequency distribution for a two-dimension case.

Analytic Hierarchy Process

Analytic Hierarchy Process (AHP) is one of the most popular methods for group decision making used in project selection. AHP simplifies complex problems by arranging the decision factors in a hierarchical structure [4]. The AHP method is consisted of the following steps [4, 5]: 1) Define the problem and determine its goal. 2) Structure the hierarchy with the decision-maker's objective at the top with the intermediate levels capturing criteria on which subsequent levels depend and the bottom level containing the alternatives. 3) Construct a set of $n \times n$ pair-wise comparison matrices for each of the lower levels with one matrix for each element in the level immediately above. The pair-wise comparisons capture a decision maker's perception of which element dominates the other. 4) There are $n \times \left((n-1)/2\right)$ judgments required to develop the set of matrices in step 3. Reciprocals are automatically assigned in each pair-wise comparison. 5) The hierarchy synthesis function is used to weight the eigenvectors by the weights of the criteria and the sum is taken over all weighted eigenvector entries corresponding to those in the next lower level of the hierarchy.

Tabular method

Tabular method (TM) is a decision-making method, which can be easily used for choosing from a large number of alternatives [1]. The TM is consisted of the following steps: 1) Create a table with columns related to the criteria, and rows related to alternatives involved into the decision-making process. 2) For each column (criterion) of the created table, put a set of alternatives in order from most to least desirable. 3) Delete from the table each row related to non-Pareto-optimal solutions. 4) Impose constraints on each of the criteria (columns), namely the worse-case values, which can be acceptable for each of the criteria, should be chosen. 5) Check if exist non-empty set of solutions (alternatives), which satisfy imposed constraints. 6) In the case of need repeat steps 4 and 5 in order to obtain a feasible set of solutions.

Decision making approach

The decision making approach proposed in this study is based on a simultaneous utilization for a given set of Pareto-optimal solutions the AHP and TM methos. It is well-known that a major drawback of the AHP is that a large number of pair-wise comparisons are needed to obtain final solution [7], therefore in the proposed approach the TM method is used in order to reduce the number of the initial Pareto-optimal solutions (the TM method allows doing it relatively easily and rapidly), and the AHP method is utilized for choosing the best alternative among the subset of reducing Pareto-optimal solutions. The decision making software packages "MPRIORITY" [8, 9] and "T-CHOICE" [8, 10] based on the AHP and TM methods, respectively, were utilized for choosing the best alternative among the obtained set of Pareto-optimal solutions. The aggregating functions approach, adaptive random search algorithm coupled with penalty functions approach, and the finite difference method with cubic spline approximation were utilized in this study to compute the initial set of the Pareto-optimal solutions.

"MPRIORITY" software

Borland C++ Builder 6.0 was used to design the "MPRIORITY" software (Fig. 5). "MPRIORITY" contains all required graphic user interface (GUI) dialogues for making the AHP's decision making process easy and quickly.

"T-CHOICE" software

Borland C++ Builder 6.0 was used to design the "T-CHOICE" software (Fig. 3 and 4). "T-CHOICE" contains all required GUI-dialogues for making the TM's decision making process easy and quickly.

Multi-objective optimization of thermal processing

In this work, the food quality factors of thiamine content and texture retention of pork puree were considered as particular objective functions [1]. The last chosen particular objective is the thermal process time; therefore, the following multi-objective optimization of the thermal process optimization problem considered in this study was:

$$\langle \Phi_1(u), \Phi_2(u), \Phi_3(u) \rangle \to \min_{u \in U}, \tag{6}$$

subject to:

$$F_0(t_f) \geq F_0^d,\ \Phi_1(u) \geq C_1^d,\ \Phi_2(u) \geq C_2^d,\ T^l \leq \Phi_3(u) \leq T^r,$$

where U is the domain of control variables u_i, $i \in 1:(N_p - 1)$, Φ_1 is thiamine retention multiplied by –1, Φ_2 is texture retention multiplied by –1, Φ_3 is thermal processing time, C_1^d, C_2^d are desired retention values and T^l and T^r are left and right limits of the process time, respectively.

RESULTS & DISCUSSION

The Pareto-optimal solutions of the multi-objective thermal processing optimization problem (6) were computed by utilizing each of the aggregating functions (2)-(5) and the Variable Retort Temperature profiles [1]. Tables 1 present the twenty Pareto-optimal solutions obtained for thermal processing.

Table 1. Obtained pareto-optimal solutions of the multi-objective thermal processing optimization problem (6).

Process Time (min)	Thiamine Retention (%)	Texture Retention (%)
111	58	47
100	57	46
76	52	47
71	50	47
60	45	45
52	40	44
47	38	42
112	59	45
91	56	47
85	55	47
69	49	48
65	47	47
61	45	46
50	39	43
106	58	45
90	55	48
80	53	47
76	51	48
56	45	43
49	38	43

The minimum number of AHP's pair-wise comparisons necessary to choose the best multi-objective thermal processing alternative among the twenty Pareto-optimal solutions presented in Table 1 is equal to 543, which of course is a large enough number of comparison for direct implementation of the AHP method. Therefore, the TM method, which is not so critical to the numbers of initial alternatives and criteria, was used initially in order to reduce the number of initial alternatives. Fig. 3 shows results obtained by "T-CHOICE" software or TM method. As we can see from Fig. 3 the criteria constraints $\Phi_1 \geq 50$, $\Phi_2 \geq 47$ and $\Phi_3 \leq 76$ were imposed on the first, second and third criterion, which means that the thermal process with the processing time to be less than 76 min., and final product with thiamine and texture retentions to be higher than 50% and 47%

respectively, will be desired for food engineer or expert. Fig. 4 shows alternatives, which satisfy the imposed constraints. Thus, the number of initial alternatives was reduced from twenty to three. In this case the minimum number of AHP's pair-wise comparisons necessary to choose the best thermal processing alternative among the three Pareto-optimal solutions presented in Fig. 4 is equal to 12.

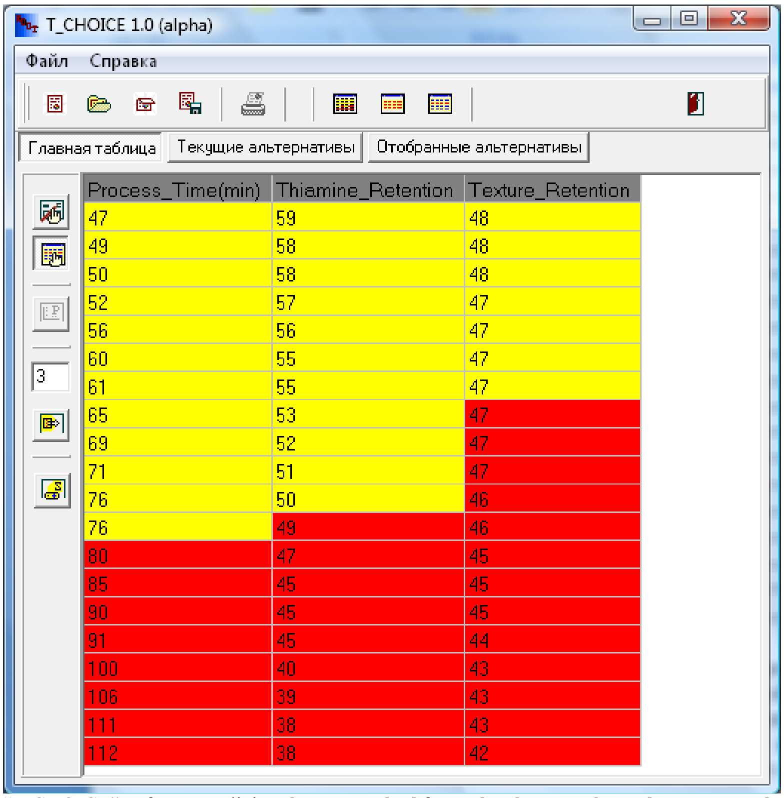

Figure 3. "T-CHOICE" software realizing the TM method for multi-objective thermal processing alternatives.

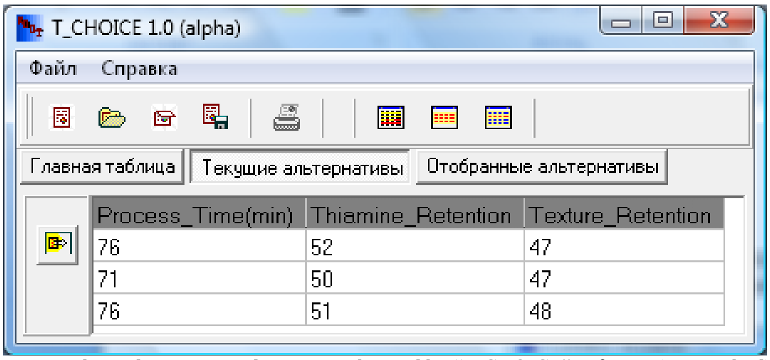

Figure 4. Thermal processing alternatives obtained by "T-CHOICE" software (TM method).

AHP model (hierarchy) related to the problem of choosing the best multi-objective thermal processing alternative among the three obtained Pareto-optimal solutions is presented on Figure 5. All required by presented hierarchy pair-wise comparison were done, and the final priorities of each thermal processing alternative were computed by "MPRIORITY" software (see Table 2). As we can see from Table 2 the process 2 is chosen as the best one.

Table 2. Thermal processing alternatives and computed priority values.

N	Process Time	Thiamine Retention	Texture Retention	Priority value
1	76	52	47	0.2259
2	71	50	47	0.5706
3	76	51	48	0.2034

CONCLUSION

The proposed in this study decision making approach and tools for solving the multi-criteria decision making problems arising in the food engineering was successfully tested on the thermal food processing problem. The utilization of TM method in the proposed approach allows utilizing the AHP method in a straightforward manner, which avoids the information overload, pair-wise comparison's routine and makes the decision making process easier. It should be noted that the set of particular criteria used in this research cannot be considered to be unique, and, depending on a practical situation, this set can be changed, and the processing profiles can be re-computed. The proposed decision making approach and software packages are useful for food scientists (research and education) and engineers (real food engineering decision making problems).

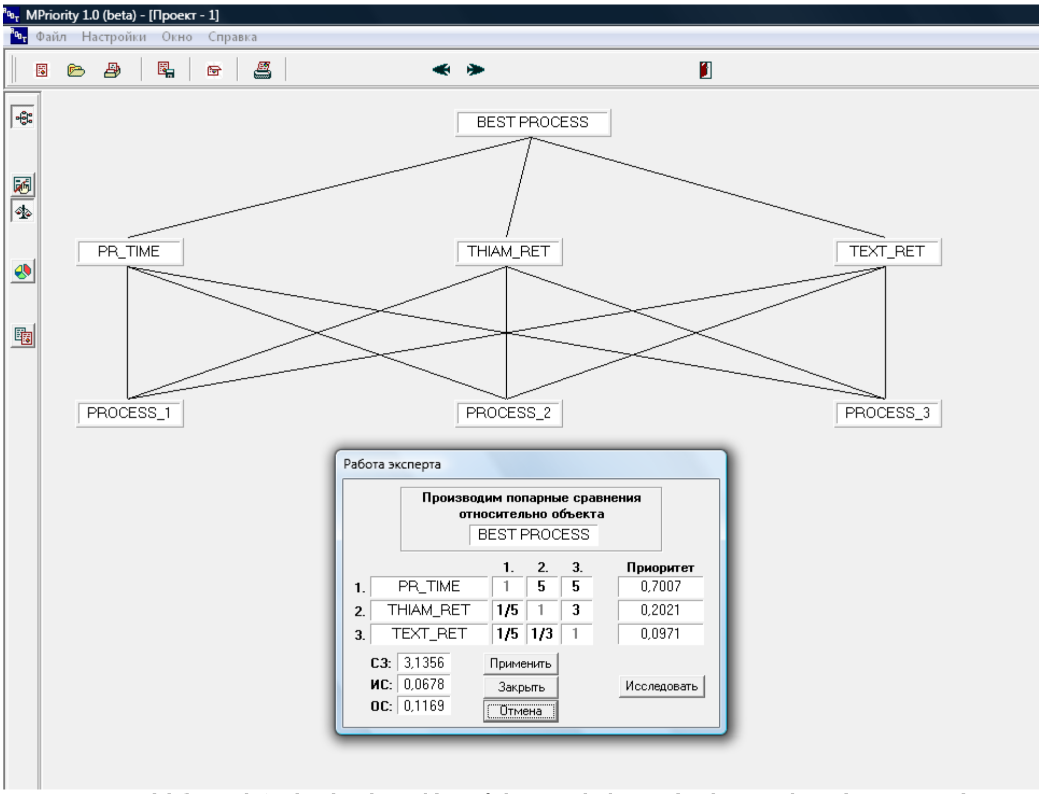

Figure 5. AHP model (hierarchy) related to the problem of choosing the best multi-objective thermal processing alternative.

REFERENCES

[1] Abakarov A, Sushkov Yu, Almonacid S., Simpson R. 2009. Multi-objective optimization based on adaptive random search method: optimization of food processing. Journal of Food Science. 74(9), E471 - E487.

[2] Sendin, J. O. H., Alonso, A. A., Banga, J. R. 2010. Efficient and robust multi-objective optimization of food processing: A novel approach with application to thermal sterilization. Journal of Food Engineering. 98(3),317-324.

[3] Seng C, Rangaiah S. 2008.Multi-objective optimization in food engineering. Chapter 4 in Taylor & Francis Book "Optimization in Food Engineering," Ed. Ferruh Erdogdu. 800 p.

[4] Pirdashti M., Mohammadi M. 2009. Multi-criteria decision-making selection model with application to chemical engineering management decisions. World Academy of Science, Engineering and Technology. 49, 54-59.

[5] T. L. Saaty T. L. How to make a decision: the analytical hierarchy process. European journal of operational research. North-Holland, 1990, 48, pp.9-26.

[6] Sushkov Yu. 1984. Multiobjective optimization of multi-regime systems. Digital System Architecture. Moscow, Moscow State University, pp. 21-24.

[7] F. J. Carmone F.J, Kara A., Zanakis S.H. A Monte Carlo investigation of incomplete pairwise comparison matrices in AHP. European Journal of Operational Research, Volume 102(3):538–553, 1997.

[8] Abakarov A. ToMakeChoice. 11 Nov. 2006. < http://tomakechoice.com/program.html>

[9] Abakarov A., Sushkov Yu. 2005. Russian Agency for Patents and Trademarks (ROSPATENT) Certificate of official registration of computer program system "MPRIORITY" № 2005612330.

[10] Abakarov A., Sushkov Yu. 2008. Russian Agency for Patents and Trademarks (ROSPATENT) Certificate of official registration of computer program system "T-CHOICE" № 2008614589.